SPACE: From CONTAINER to COHESION and QUALITY

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What is SPACE? “A space is an object in a category of spaces” (Lawvere, 2015). This answer in turn raises questions such as ‘what is category?’ Before we go further along these lines, let us look at our naïve notion of space. Space is a container … “destined to be inhabited by the accidents of matter” (Cartier, 2001); it’s an arena of change, of quantitative variation, and of becoming (Lawvere, 1992). Simply put, space is like a blank slate used to draw figures of all sorts. This ancient idea of space embodies a profound dialectic: container vs. contents, which continues to fuel the ongoing refinement of the concept of space. Containers (e.g. rooms) are made of locations or positions where contents (tables and chairs) can be placed. With positions objectified as points, and with the added realization that positions can be near or far apart, we arrive at an understanding of space as a collection or set of points with some additional structure. These sets equipped with structure are categories and a category is a mathematical universe of discourse (Lawvere and Rosebrugh, 2003, pp. 239-240; Lawvere and Schanuel, 2009, pp. 135-151). The constituents of space i.e. points were initially thought of as having no structure, like the elements of a set. Subsequently, points were recognized as an objectification of constancy—a limiting case of variation (Lawvere, 1975). From this relational perspective, points are not that different from figures, albeit special. It is possible, with points considered as figures of special shape, to have spaces without their building blocks (points; Lawvere and Schanuel, 2009, pp. 230-235). More importantly, the geometry of a space is determined by figures [and their
incidences] within the space (Lawvere and Schanuel, 2009, pp. 370-371). In other words, the container is determined by its contents, which is reminiscent of the content-addressable memories of neural networks (Hopfield, 1982). This paradigm shift—from space as a container existing in-and-of itself to space as a structure of the relations between its contents—mirrors a parallel development in physics (Roy, 1998, pp. 222-229).

The reach of the figural-turn of the mathematical study of space extends far beyond space—all the way to bridging the abyss between space and symbol. With symbolic logic as the algebra of spatial parts (Lawvere and Rosebrugh, 2003, pp. 193-212), we have rudiments of a scientific account of how the commonplace experience of ‘thinking about space’ works: “Human activities such as building a house, navigating the solar system, require plans that can work. Planning any such undertaking requires the development of thinking about space. Each development involves many steps of thought and many related geometrical constructions on spaces. Because of the necessary multistep nature of thinking about space, uniquely mathematical measures must be taken to make it reliable. Only explicit principles of thinking (logic) and explicit principles of space (geometry) can guarantee reliability. Category theory permitted making the principles of logic and geometry explicit by discovering the common form of logic and geometry. The principles of both logic and geometry rest on “naturality” of the transformations between spaces and the transformations within thought.” (Picado, 2007).

A fascinating recent development in the mathematical study of space is the recollection of Maxwell’s deep insight: description of the nature of space depends on the needs of investigation. In studying the resultant categories of space (e.g. combinatorial, smooth), it became clear that COHESION is the defining attribute of space (Lawvere, 1994a, 2007; Lawvere and Rosebrugh, 2003, p. 232). Cohesive spaces, in contrast to discrete spaces such as sets, are
characterized by a truth value object that allows true to become false (Grothendieck, 1983; Lawvere, 2003). Moreover, with quality as that which remains upon identifying all quantitative variations (Lawvere, 1992), qualities of cohesive spaces are defined (Lawvere, 2008). The essence or theory of a space is an intensive quality, while the form or logic of a space is an extensive quality. The axiomatization of cohesion can be used to formalize the unity or cohesiveness of conscious experience (Roskies, 1999); while the definition of quality can be used to formalize the qualitative universals given in particular conscious experiences i.e. qualia (Lewis, 1929, p. 121).

Within mathematical experience, the study of space gave rise to various concepts (e.g. topos; Lawvere, 2014; Lawvere and Rosebrugh, 2003, pp. 245-247), and the thus abstracted theories (e.g. dynamical evolution) are in turn represented as spaces (Lawvere, 2002; Lawvere and Rosebrugh, 2003, pp. 154-155). This productive interplay between spaces and their qualities (theory and broad objective logic; Lawvere, 1994b, 1996, 2004; Lawvere and Rosebrugh, 2003, pp. 235-236, 239-240) moves mathematics towards ever more faithful reflections of space in particular and reality in general.

References


